# Generalized Dihedral Automorphic Loop and its Half-isomorphisms

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# Inner mapping group

Let $L$ be a loop and $x \in L$. We define the left and right translations of $x$ in $L$, respectively by:

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The *inner mapping group* of $L$ is defined by:

$$Inn(L) = \{\varphi \in Mlt(L) \mid (1)\varphi = 1\}$$

# Automorphic Loop

A loop $L$ is called *automorphic loop*, or *A-loop*, if all elements of $Inn(L)$ are automorphisms of $L$.

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# Automorphic Loop

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In the paper The Structure of Automorphic Loops[1], the authors constructed a type of automorphic loop from a group. They called it *generalized dihedral automorphic loop*.

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# Generalized Dihedral Automorphic Loop

Let $G$ be a finite abelian group and $\alpha \in Aut(G)$. Define $Dih(\alpha, G) = \mathbb{Z}_2 \times G$ with the following operation:

$$(i,u) * (j,v) := (i+j, \alpha^{ij}(u^{(-1)^j} v)) \quad i,j \in \mathbb{Z}_2, u,v \in G$$

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or, for $u, v \in G$:

$$\begin{gathered}(0,u) * (0,v) = (0,uv)\\ (0,u) * (1,v) = (1,u^{-1}v)\\ (1,u) * (0,v) = (1,uv)\\ (1,u) * (1,v) = (0,\alpha(u^{-1}v))\end{gathered}$$

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# Generalized Dihedral Automorphic Loop

In the same paper [2], the authors proved that $Dih(\alpha, G)$ is an automorphic loop.

It is easy to see that if $\alpha = I_d$, then $Dih(\alpha, G)$ is a group, and vice versa.

Also, $Dih(\alpha, G)$ is commutative if, and only if, $G$ has period 2.

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# Half-Isomorphism

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A *half-isomorphism* of $(L, *)$ into $(L', \cdot)$ is a bijection $f : L \longrightarrow L'$

$$f(x * y) = f(x) \cdot f(y)$$
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A half-isomorphism is *trivial* if it is either an isomorphism or an anti-isomorphism.

# Questions

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Yes. Using the software GAP, we found many examples of non trivial half-isomorphisms between generalized dihedral automorphic loops of order 6, 8, 10, etc.

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Question 4. What form do these non trivial half-isomorphisms take?

# Our Results

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Lemma 1. Let $(L, *), (L', \cdot)$ be loops and $f : L \to L'$ a half-isomorphism. If $L'$ is commutative, then $L$ is commutative and $f$ is an isomorphism.

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From now on we assume that $G$ does not have period 2.

Lemma 2. Let $(L, *), (L', \cdot)$ be $A$-loops, $f : L \to L'$ a half-isomorphism and $x \in L$ such that the order of $x$, denoted by $o(x)$, is finite. Then $o(f(x)) = o(x)$.

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Proposition 3. The function $f'$ defined above is an automorphism of $G$.

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Proposition 4. Let $f : Dih(\alpha, G) \to Dih(\beta, G)$ be a half-isomorphism. Then there exists $a \in G$ such that for all $u \in G$, we have:

$$f((i,u)) = \begin{cases} (0, f'(u)), & i = 0 \\ (1, af'(u^{\epsilon_u})), & i = 1 \quad (\epsilon_u \in \{-1, 1\}) \end{cases}$$

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Let $f' : G \to G$ be an automorphism and $a \in G$. Define $f_{-a}, f_{+a} : Dih(\alpha, G) \to Dih(\beta, G)$ by

$$f_{-a}((i,u)) = \begin{cases} (0, f'(u)), & i = 0 \\ (1, af'(u^{-1})), & i = 1 \end{cases}$$

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Since $G$ does not have period 2, $f_{-a} \neq f_{+a}$, for all $a \in G$.

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Proposition 5. Let $f' : G \to G$ be an automorphism and $a \in G$. Then:

a) If $f'\alpha = \beta f'$, then $f_{+a}$ is an isomorphism and $f_{-a}$ is an anti-isomorphism.
b) If $f'\alpha = \beta f' J$, then $f_{+a}$,$f_{-a}$ are non trivial half-isomorphisms, where $J$ is the inversion map.

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b) If $f'\alpha = \beta f' J$, then $f_{+a}$,$f_{-a}$ are non trivial half-isomorphisms, where $J$ is the inversion map.

Proposition 6. Let $f : Dih(\alpha, G) \to Dih(\beta, G)$ be a half-isomorphism. If $f'\alpha \in \{\beta f', \beta f' J\}$, then $f \in \{f_{-a}, f_{+a} \mid a \in G\}$.

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a) $f : Dih(\alpha, G) \rightarrow Dih(\beta, G)$ is a non trivial half-isomorphism.

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Theorem. The statements are equivalent:

a) $f : Dih(\alpha, G) \to Dih(\beta, G)$ is a non trivial half-isomorphism.

b) $f \in \{f_{-a}, f_{+a} \mid a \in G\}$, with $f' : G \to G$ automorphism and $f'\alpha = \beta f' J$.

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The sketch of proof:

(b) => (a) It follows by proposition 5.

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$H = \{v \in G \mid f'\alpha(v) = \beta f'(v)\} \leq G$

$K = \{v \in G \mid f'\alpha(v) = \beta f'J(v)\} \leq G$

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$=> G = H \cup K$

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$\overset{\text{Proposition 6}}{=>} f \in \{f_{-a}, f_{+a} | a \in G\}$

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Since $f$ is a non trivial half-isomorphism, by Proposition 5 we have $f'\alpha = \beta f' J$.

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Corollary 1. Let $G$ be a group such that period is not 2. Then there exists a non trivial half-isomorphism of $Dih(\alpha, G)$ into $Dih(\beta, G)$ if, and only if, $\alpha$ is conjugated to $\beta J$ in $Aut(G)$.

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Corollary 2. Let $G$ be a group such that period is not 2 and $\alpha, \beta$ automorphisms of $G$ such that $\alpha$ is conjugated to $\beta J$. Then there are $2|G||C_{Aut(G)}(\alpha)|$ non trivial half-isomorphisms of $Dih(\alpha, G)$ into $Dih(\beta, G)$.

Where $C_{Aut(G)}(\alpha) = \{\psi \in Aut(G) \,|\, \alpha\psi = \psi\alpha\}$

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# Example

Consider $G = C_3$. Then $Dih(I_d, C_3)$ is the dihedral group of order 6 and $Dih(J, C_3)$ is the smallest non associative automorphic loop.

| $*$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 2 | 3 | 1 | 6 | 4 | 5 |
| 3 | 3 | 1 | 2 | 5 | 6 | 4 |
| 4 | 4 | 5 | 6 | 1 | 2 | 3 |
| 5 | 5 | 6 | 4 | 3 | 1 | 2 |
| 6 | 6 | 4 | 5 | 2 | 3 | 1 |

| $\cdot$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 |
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| 5 | 5 | 6 | 4 | 3 | 1 | 2 |
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The non trivial half-isomorphisms between $Dih(I_d, C_3)$ and $Dih(J, C_3)$ are:

$$I_{d\ -1} = (5\ 6) \qquad I_{d\ -2} = (4\ 5) \qquad I_{d\ -3} = (4\ 6)$$

$$I_{d\ +1} = I \qquad I_{d\ +2} = (4\ 5\ 6) \qquad I_{d\ +3} = (4\ 6\ 5)$$

# Example

The group $C_3 = \{1, 2, 3\}$ has two automorphisms: $I_d$ and $J$.

The non trivial half-isomorphisms between $Dih(I_d, C_3)$ and $Dih(J, C_3)$ are:

| | | |
|---|---|---|
| $I_{d\ -1} = (5\ 6)$ | $I_{d\ -2} = (4\ 5)$ | $I_{d\ -3} = (4\ 6)$ |
| $I_{d\ +1} = I$ | $I_{d\ +2} = (4\ 5\ 6)$ | $I_{d\ +3} = (4\ 6\ 5)$ |
| $J_{-1} = (2\ 3)$ | $J_{-2} = (2\ 3)(4\ 5\ 6)$ | $J_{-3} = (2\ 3)(4\ 6\ 5)$ |
| $J_{+1} = (2\ 3)(5\ 6)$ | $J_{+2} = (2\ 3)(4\ 5)$ | $J_{+3} = (2\ 3)(4\ 6)$ |

# Thank you for your attention!

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